

**Porosity Correction and Effective Thermal Conductivity of
Real Two-Phase Systems.**

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**Paper to be presented at the 14th Symposium on Thermophysical Properties, June
25-30, 2000, Boulder, Colorado, U.S.A.**

ABSTRACT

It is observed that theoretical expressions based on rigid geometry do not represent the true state of affairs of a real two-phase system. Therefore, the values of effective thermal conductivity (ETC) predicted by these models deviate from experimental results. Keeping this in mind, a theoretical model developed earlier by us is used for two-phase porous systems assuming linear flow of heat flux lines having regular three-dimensional simple cubic geometry. To take account of non-linear flow of heat flux lines in real systems, incorporating an empirical correction factor in place of physical porosity modifies an expression for ETC. An effort is made to correlate it in terms of the ratio of the thermal conductivities of the constituents and the physical porosity. Theoretical expression so obtained for correction in physical porosity has been tested on a large number of samples cited in the literature and found that the values predicted are quite close to the experimental results.

KEY WORDS: Effective thermal conductivity; heat flux; porosity correction term; porous systems; physical porosity; non-linear flow

1. INTRODUCTION

The theoretical modelling for two-phase dissimilar systems of industrial importance is a challenging task for engineers and physicists, and of major interest to soil scientists and geologists. It is required because of increasing use of porous materials as insulating envelopes in solar ponds, non-conventional refrigerators, air conditioners and high temperature furnaces. The study of thermal parameters of these two-phase systems is also valuable for the explosive industry, the ceramics industry, nuclear reactors and in missile technology. The ETC depends on various factors such as thermal conductivity, porosity, size of particles and packing of the constituent phases. Accounting of all these factors in order to predict ETC is a complex affair. In the literature one finds several efforts [1 to 6] in which the situation has been simplified by assuming that the particles are of specific shape and arranged in a particular geometry with the continuous phase.

The present paper uses the theoretical model proposed by us [7] to predict ETC of dissimilar two-phase porous systems with cubic inclusions. However, in real systems the packing and the shape of the particles are random. In order to incorporate varying individual geometry and non-linear flow of heat flux lines generated by the difference in thermal conductivities of constituent phases, a correction term in place of the physical porosity has been introduced. In the literature similar attempts have been made [8 to 12] but for a limited ETC ratio. Expression for porosity correction term has been obtained empirically by simulating experimental data reported in the literature. The present approach is simple and provides wider applicability to cubical model and enhances its ability to predict correctly the ETC of two-phase porous systems and systems having high ratios of thermal conductivity of their constituent phases.

2. THEORY

The authors [7] have applied a resistor model to obtain the following expression for predicting ETC of two-phase systems with cubic inclusions

$$\lambda_e = \frac{\lambda_f \left[(\lambda_s - 2\lambda_f) \phi_s^{1/3} + \lambda_f \right]}{\left[(\lambda_s - 2\lambda_f) \phi_s^{1/3} - (\lambda_s - 2\lambda_f) \phi_s^{2/3} + \lambda_f \right]} \dots(1)$$

Where λ_s and λ_f are the thermal conductivities of the solid and fluid phases respectively and ϕ_s is the fractional volume of the solid phase.

It is observed that theoretical expressions based on rigid geometry do not represent the true state of affairs of a real two-phase porous system. The ETC depends upon various characteristics of the system. The most prominent amongst them being the volume fraction and thermal conductivity of the constituent phases. Thus, for practical utilization, we have to modify this expression by incorporating some correction term. Tareev [13] has shown that, during the flow of electric flux from one dielectric to another dielectric medium, the deviation of flux lines in any medium depends upon the ratio of the dielectric constants of the two media. By the analogy we can have the concentration of thermal flux altered from its previous value as it passes through another medium and that the amount is a function of λ_s/λ_f . Such a deviation causes a zig- zag path of the flux lines in the bulk and also alters the density of flux lines in the constituent phases. The concentration of flux lines is greater in the phase of higher conductivity then it is in the phase of lower conductivity. If the flow of flux lines is linear then this porosity function would have been numerically equal to the physical

porosity of the sample. In cases where curvature in the flow lines occurs, the porosity function will not be equal to the physical porosity of the sample but it should be a function of the ratio of the thermal conductivities of the constituent phases as well as of the physical porosity of the sample. Considering random packing of the phases, non-uniform shape of the particles and flow of heat flux lines not restricted to be parallel we here replace physical volume fraction of the solid phase ϕ_s by porosity correction term F_p . F_p in general should be a function of the physical volume fraction of the solid phase and the ratio of thermal conductivities of the constituent phases. Therefore, expression (1) may be written as

$$\lambda_e = \frac{\lambda_f \left[(\lambda_s - 2\lambda_f) F_p^{1/3} + \lambda_f \right]}{\left[(\lambda_s - 2\lambda_f) F_p^{1/3} - (\lambda_s - 2\lambda_f) F_p^{2/3} + \lambda_f \right]} \dots(2)$$

3. RESULTS AND DISCUSSION

Table 1 cites the experimental results of ETC and other data for two-phase porous systems reported in the literature. Without any correction term equation (1) exhibits large deviations from the experimental results. The porosity correction term introduced for each sample has been computed using equation (2) and data reported in table 1. Because F_p is a function of ϕ_s and λ_s / λ_f , we have made an effort to correlate it in terms of ϕ_s and the ratio of thermal conductivities of the constituent phases. Keeping this in mind, we have tried many combinations. Such plots of $F_p/\phi^{1/3}$ versus $\log(\lambda_s/\lambda_f)$ are shown in figures 1 and 2. It can be observed from the graphs that F_p increases

roughly linearly with increasing $\log (\lambda_s / \lambda_f)$ up to a certain value. After that, $F_p/\phi^{1/3}$ drops to a certain extent and further increases with increasing $\log(\lambda_s/\lambda_f)$ in the case of metallic powders. We have used curve fitting technique and found that the expression

$$C = C_1 + C_2 \log (\lambda_s / \lambda_f) \quad \text{..... (3)}$$

Best fitted the curves shown in figures 1 and 2. Here $C_1 = 0.3262$ and $C_2 = 0.2926$ for $1 < \lambda_s/\lambda_f < 500$ and $C_1 = -0.3092$ and $C_2 = 0.3644$ for $1000 < \lambda_s/\lambda_f < 15000$.

Thus for cubical model used here, the appropriate correction term is found to be

$$F_p = \phi^{1/3} [C_1 + C_2 \log (\lambda_s/\lambda_f)] \quad \text{..... (4)}$$

On inspection of the above equation and experimental results reported in the literature, it is found that this porosity correction term gives better results for two-phase porous systems when the porosity is in the range 0.2 – 0.8.

Table 2 gives a comparison of experimental values of ETC and values calculated using equations (2) and (4). We found that the values obtained agreed well with the experimental results.

4. CONCLUSIONS

The porosity of any system is a measurable quantity, but in models based on rigid geometry, this quantity must be replaced by a porosity correction term. It is due to non linear flow of heat flux lines in two phase dissimilar systems that causes one to

query the concept of a simple volumetric average of the phases. On the other hand, if the flux lines are linear then the physical porosity will be equal to the actual porosity of the system and a simple weighted average can be taken. This saved the effort that would have been required to determine an additional parameter without compromising on the accuracy of the results.

ACKNOWLEDGEMENT

Author is thankful to Dr. D.R. Chaudhary and K.J. Singh for their valuable suggestions and discussion.

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Table-1. Experimental values of ETC and other data of two-phase systems (values of thermal conductivity are in W / m K)

S.No.	Sample Solid / fluid	Porosity of solid phase	Th. Cond. of solid	Th. Cond. of fluid	ETC exp.
1	Glass/IC8[14]	.57	1.061	.144	.406
2	Glass/benzene[14]	.65	1.201	.14	.50
3	Silica/water[14]	.56	12.414	.586	2.544
4	Glass/air[14]	.60	1.091	.029	.180
5	Glass/air[14]	.65	1.201	.028	.220
6	Glass/air[14]	.60	1.13	.026	.190
7	Microbeads/air	.65	1.046	.026	.193
8	Leadshots/water	.62	34.347	.627	5.404
9	Lead/water[17]	.60	33.764	.586	4.329
10	Zircona/air[18]	.58	2.001	.030	.230
11	Zircona/air[18]	.64	2.001	.030	.281
12	Steel/eth.alcohol	.449	20.864	.337	2.009
13	Desert sand/air	.615	3.36	.026	.387

14	Desert sand/air	.595	3.36	.026	.336
15	Desert sand/air	.561	3.36	.026	.312
16	Desert sand/air	.550	3.36	.026	.289
17	Miami siltloam/air	.55	2.932	.023	.221
18	Alumina/air[14]	.274	28.2	.026	.133
19	Iron/air[14]	.42	65.4	.026	.217
20	Copper/air[15]	.18	398	.027	.232
21	Copper/air[15]	.22	398	.027	.282
22	Copper/air[15]	.25	398	.027	.317
23	Copper/air[15]	.28	398	.027	.371
24	Copper/air[15]	.35	398	.027	.546
25	Aluminum/air	.386	218.0	.026	.356
26	Brass/air[15]	.62	109.2	.026	.514
27	Brass/air[15]	.50	109.2	.026	.378
28	Brass/air[15]	.45	109.2	.026	.348

Table-2 Experimental values of ETC of two-phase systems listed in table 1

compared with theoretical values obtained using values of F_p

from equation (4) (values of thermal conductivity are in W / m K)

Sample No.	Porosity of solid phase	ETC		Error (%)
		Exp.	Theo.	
1	.57	.406	.392	3.4
2	.65	.500	.432	13.7
3	.56	2.544	2.775	9.1
4	.60	.18	.187	3.9
5	.65	.22	.204	7.5
6	.60	.19	.180	5.3
7	.65	.193	.183	5.3
8	.62	5.404	4.985	7.8
9	.60	4.329	4.658	7.6
10	.58	.23	.249	8.5
11	.64	.281	.272	3.3
12	.449	2.009	2.251	12

13	.615	.387	.331	14.4
14	.595	.336	.318	5.5
15	.561	.312	.296	5.2
16	.55	.289	.289	0.0
17	.55	.221	.254	14.8
18	.274	.133	.131	1.4
19	.42	.217	.228	4.9
20	.18	.232	.226	2.6
21	.22	.282	.271	3.8
22	.25	.317	.312	1.7
23	.28	.371	.360	3.0
24	.35	.546	.521	4.6
25	.387	.356	.397	11.7
26	.62	.514	.536	4.3
27	.50	.378	.366	3.1
28	.45	.348	.318	8.7

FIGURE CAPTIONS

Figure 1 The variation of $C = Fp / \phi^{1/3}$ with $\log(R) = \log(\lambda_s / \lambda_f)$. Symbols are experimental data and the full line is the theoretical simulation ($1 < \lambda_s / \lambda_f < 500$).

Figure 2 The variation of $C = Fp / \phi^{1/3}$ with $\log(R) = \log(\lambda_s / \lambda_f)$. Symbols are experimental data and the full line is the theoretical simulation ($1000 < \lambda_s / \lambda_f < 15000$).



